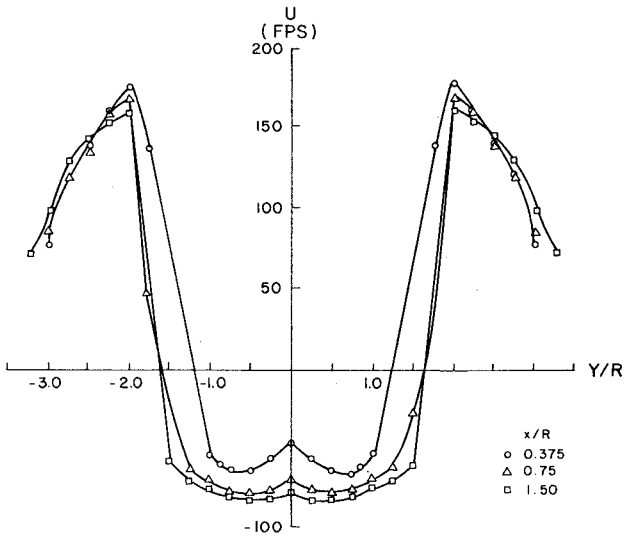

Fig. 5 Mean velocity profiles, 2-in.-diam disk.

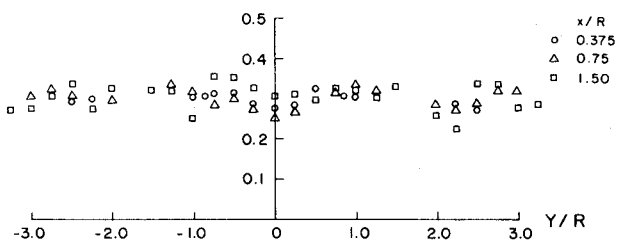

Fig. 6 Turbulent intensity profile summary, 2-in.-diam disk.

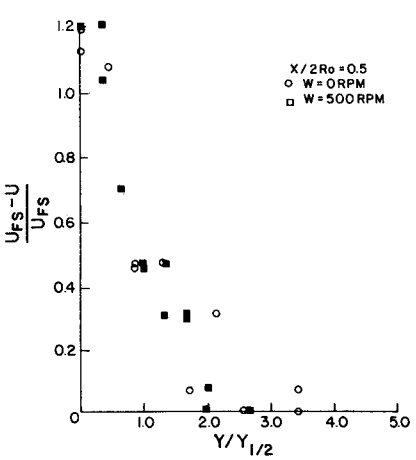

Fig. 7 Wake velocity defect profile, $X/2R_0 = 0.5$.

scheme due to its extreme sensitivity to scattered light. The background flare light and resultant photon pileup cause a skewed and/or distorted photon correlation curve. In an attempt to alleviate this source of error, a data algorithm is proposed and evaluated.

Unsteady (Flapping) Jet

Although the data is not shown in this report, a nozzle fitted with a fluidic feedback loop is oscillated from 4 to 18 Hz. The resultant mean velocities are measured both at the centerline of the flowfield ($Y/D=0$) and outside the lip of the nozzle ($Y/D=1.5$) for various downstream locations ($X/D=1$ to 5). The photon scheme is sensitive to the relatively low frequency oscillations. One interesting observation is that at $Y/D=0$, the mean velocity is actually the largest in the magnitude at $X/D=5$.

Summary

The photon correlation technique is used for several different turbulent fluid investigations. The different results

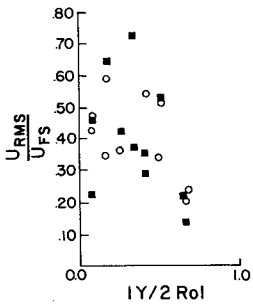

Fig. 8 Turbulent intensity profile, $X/2R_0 = 0.5$.

point to significant advantages and disadvantages for the photon resolved processor. The main advantages are:

1) Using only naturally occurring contaminant, credible mean velocity measurements can be made in flow configurations which would be inaccessible using the counter and/or tracker techniques.

2) Mean velocity information is not strongly dependent on incident laser intensity, while turbulent intensities are significantly affected.

The main disadvantages are:

1) There is no control over particle size distribution of light scatters.

2) Actual experimental time required for photon resolved autocorrelation curve may be considerable.

3) Time-dependent velocity information is not available.

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Calculation of Turbulent Diffusion Flame Using the Coherent Flame Sheet Model

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Introduction

ONE of the difficulties in the computation of combustion flowfields is the modeling of the chemical reaction terms.

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This difficulty arises because the flow variables used in the computation are averaged over the turbulent fluctuations. Since the chemical reaction terms involve functions of the species concentrations and temperature, averaging the reaction terms would lead to the usual closure problem of unknown higher moments of the variables. As measurements of these moments are extremely difficult, an effective reaction rate based on the averaged variables is usually employed. As a result, there is considerable uncertainty in modeling the chemical reaction terms in this manner. A number of efforts have been mounted to improve this aspect of modeling. For example, the effective reaction rate has been calculated from probability density function (pdf) of the flow variables.[1-4] Another approach is to model the structure of the turbulent flame explicitly so that the chemical reaction terms can be accounted for by summing the reaction in the flame structures.[5,6] The advantage of such a theory lies in the decoupling of the chemical reaction from the fluid mechanics. The chemical reaction is treated as a tabulation of precalculated results from a one-dimensional stretched flame calculation. In this Note, the coherent flame sheet model is formulated in the context of the k-ϵ model and results of a series of one-dimensional methane/air stretched flame calculations are presented.

The Coherent Flame Sheet Model

In a turbulent diffusion flame at high Reynolds number, the chemical reaction rate is fast compared to the mixing rate and the overall combustion is mixing controlled. The actual combustion takes place as thin laminar diffusion flames at the interface of the unmixed fuel and air pockets that have length scales similar to the turbulent eddies. The combustion process may be visualized as an ensemble of these laminar flame sheets embedded in the strain field of the fluid. This stretched flame is kept in steady state by the balance of the generation of heat and product at the interface and the spreading of these quantities along the flame by flame stretching. Then, the chemical reaction is reduced to the calculation of that of a one-dimensional stretched flame, and the overall reaction rate is obtained by multiplying the reaction rate per unit flame surface area and the flame density Σ (amount of flame surface area per unit volume of the fluid, with dimension 1/length).

This approach may be incorporated into the k-ϵ model as follows. If $\bar{Y}_1$, $\bar{Y}_2$, and $\bar{Y}_3$ are the mass fractions (averaged) of the fuel, oxidizer, and reaction product, respectively, their transport equations are

$$L(\bar{Y}_1) = -J_1\Sigma \tag{1}$$

$$L(\bar{Y}_2) = -J_2\Sigma \tag{2}$$

$$\bar{Y}_3 = 1 - \bar{Y}_1 - \bar{Y}_2 \tag{3}$$

where L is the convective, diffusive operator with the appropriate transport coefficient as defined by the k-ϵ model. The details of this operator have been documented elsewhere.[7] The fluxes J_1 and J_2 represent the mass consumption rates of the fuel and oxidizer per unit flame surface area.

The fluxes J_i are functions of $\bar{Y}_1$, $\bar{Y}_2$, $\bar{T}$, and the flame stretching rate e (with the dimension velocity/length),

$$J_i = J_i(\bar{Y}_1, \bar{Y}_2, \bar{T}, e); \quad i = 1,2 \tag{4}$$

The flame stretching is due to the straining of the fluid by the turbulent eddies. The stretching rate e may be related to the k-ϵ model by a suitable velocity and length scale. Dimensional argument leads to

$$e = C_e\epsilon/k \tag{5}$$

For lack of a better choice, the proportional constant C_e is taken to be unity. Similarly, the enthalpy equation becomes

$$L(h) = J_1\Sigma(\text{h.v.}) \tag{6}$$

where h.v. is the heating value of the fuel.

To complete the model, a transport equation is written for Σ. Since Σ is generated by flame stretching due to the turbulent eddies, an appropriate generation rate would be $e\Sigma$. In addition, the flame sheets are destroyed when the fuel or oxidizer in a layer is used up (burned through). Since the average separation between the flame sheets ℓ is $\sim 1/\Sigma$ and the overall reaction rate is proportional to Σ, the destruction rate for Σ must be proportional to Σ^2. Therefore, the conservation equation for Σ is then

$$L\left(\frac{\Sigma}{\rho}\right) = e\Sigma - \left(\frac{J_1}{\bar{Y}_1} + \frac{J_2}{\bar{Y}_2}\right)\Sigma^2 \tag{7}$$

The variable Σ/ρ is used in order to be consistent with the definition of the operator L.

One-Dimensional Stretched Flame Calculation

The burning velocities J_i are calculated from a one-dimensional stretched flame. In general, $J_i = J_i(\bar{Y}_1, \bar{Y}_2, \bar{T}, e)$. In the flame sheet model, the chemical reaction rate is taken as infinite and the only time scale involved would be the stretching time e^{-1}. By measuring time in units of e^{-1}, we have $J_i = J_i(\bar{Y}_1, \bar{Y}_2, \bar{T})$ and the calculation need not be repeated for different values of e.

We shall assume the flame surface to lie along the x axis and define a stretching rate e as

$$e = \frac{\partial u}{\partial x} \tag{8}$$

There is no variation of all the other properties in the x direction. The continuity equation is then

$$\frac{\partial \rho v}{\partial y} = -\rho e \tag{9}$$

The conservation equations for the species and energy are

$$\rho v\frac{\partial Y_i}{\partial y} = -\frac{\partial}{\partial y}j_i; \qquad i = 1,2 \tag{10}$$

$$\rho v C_p\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\lambda\frac{\partial T}{\partial y} - \Sigma_1 j_i C_{pi}\frac{\partial T}{\partial y} \tag{11}$$

where subscripts *1*, *2*, and *3* refer to the fuel, oxidant, and product respectively, and λ is the thermal conductivity. Since, on each side of the flame interface, there is only fuel and product or oxidant and product, the mass fluxes j_i are given by the binary diffusion expressions

$$j_i = -\frac{M_3}{M}\rho D_{i3}\frac{\partial Y_i}{\partial y} \tag{12}$$

where Y is the mass fraction, M_3 the product molecular weight, and M the averaged molecular weight.

The boundary conditions are

$$T(-\infty), T(\infty), Y_1(-\infty), Y_2(\infty) = \text{given} \tag{13}$$

$$Y_1(0) = Y_2(0) = 0 \tag{14}$$

At the fuel/oxidant interface, the mass flux ratio of the fuel and oxidant should be equal to the fuel reaction ratio ϕ. (This

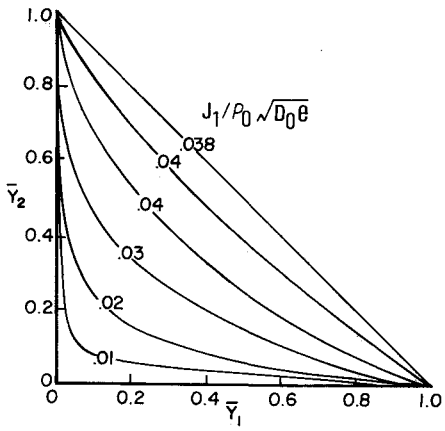

Fig. 1 Normalized fuel consumption flux J_1 as a function of the mass fraction of fuel ($\bar{Y}_1$) and air ($\bar{Y}_2$) for an adiabatic one-dimensional stretched methane/air flame.

ratio is equal to the mass of fuel needed to react with unit mass of air.) Therefore,

$$j_1(0_-) + \phi j_2(0_+) = 0 \tag{15}$$

In addition, there is a jump in the temperature gradient because of the heat of reaction. Since this discontinuity is a result of the infinite reaction rate assumption, there is a point source of heat at the interface. This jump is given by

$$-\left[\lambda \frac{\partial T}{\partial z}\right]_-^+ = j_1 \text{h.v.} \tag{16}$$

The above set of equations has been solved for an atmospheric methane/air stretched flame using the thermodynamics properties of Refs. 8 and 9. The binary diffusion coefficients D_{13} and D_{23} are taken simply as proportional to $T^{3/2}$

$$D_{i3} = D_0 (T/T_0)^{3/2} \tag{17}$$

and the thermal conductivity λ is taken to be proportional to $T^{1/2}$. The fuel/air consumption fluxes J_i are

$$J_1 = |j_1(0-)|; \quad J_2 = |j_2(0+)| \tag{18}$$

In general, J_1 is a function of $\bar{Y}_1$, $\bar{Y}_2$, and $\bar{T}$. Values of $\bar{Y}_1$ and $\bar{Y}_2$ may in the range of 0-1 and $\bar{T}$ may range from the inlet to the adiabatic flame temperature. To examine the sensitivity to these parameters, an adiabatic flame is considered. In this case, the heat release from the combustion product is distributed uniformly and without heat loss of the fuel/air/product mixture. Therefore, the specification of $\bar{Y}_1$ and $\bar{Y}_2$ (and the inlet temperature) would completely determine $\bar{T}$. A contour plot of $J_1(\bar{Y}_1, \bar{Y}_2)$ is shown in Fig. 1. The values for J_1 has been normalized by $\rho_0\sqrt{D_0 e}$, where ρ_0 is the air density at 300 K and 1 atm. For a fixed value of Y_1 (or Y_2), J_1 first increases with Y_2 (or Y_1) and then decreases. The peak values for the normalized J_1 is ~ 0.04.

To examine the sensitivity of J_1 to T, the above calculation is repeated with 30% of the heat release from the product of combustion in the bulk taken out. It was found that the resultant values of J_1 are within 5% of the corresponding values of the adiabatic case. To understand this weak dependence, it can be argued, on a dimensional basis, that

$$J_1 \sim \rho\sqrt{De} \tag{19}$$

For a constant pressure flame $\rho \sim 1/T$ and $D \sim T^{3/2}$, hence $J_1 \sim T^{-1/4}$. This T scaling is modified somewhat by the $c_p(T)$ relations, but it serves to illustrate the weak T dependence. Therefore, as a simplifying assumption, the T dependence may be dropped and the tabulation of $J_1(Y_1, Y_2)$ for the adiabatic case is sufficient as far as the heat release reaction is concerned.

Conclusion

The calculation of turbulent diffusion flame in terms of a flame sheet model has been formulated. The model is applicable when the chemical reaction is fast compared to the turbulent time scale so that combustion only occurs at thin flame sheets. In such a model, the chemical reaction term is the product of a flame density (amount of flame surface area per unit volume) and a reactant consumption rate that may be precalculated by a one-dimensional stretched flame calculation. Furthermore, the consumption rate is a weak function of the temperature, so that tabulation of the rate as a function of fuel and oxidant mass fractions for the adiabatic case is a good approximation for the heat release reaction.

Acknowledgments

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